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Ratchet potential in d.c. SQUID's and reduced two-junction interferometer models

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Abstract. For finite values of the parameter β and in the presence of small structural asymmetry and inhomogeneity of the junctions parameters, the two-junction interferometer model can be described by means of a single non-linear differential equation in such a way that a ratchet potential for SQUID's can be deduced. The resulting dynamical equation, derived for first-order corrections with respect to a symmetric and homogeneous model with $\beta = 0$, presents an additional second-harmonic term and a cosine term in the expression for the effective current-phase relation. For opportune values of the perturbation parameters a ratchet potential with pre-definite characteristics can be obtained.

PACS. 74.50.+r Tunneling phenomena; point contacts, weak links, Josephson effects – 85.25.Dq Superconducting quantum interference devices (SQUIDs)

The question on ratchet potentials in superconducting quantum interference devices (SQUID's) was first addressed by Zapata et al. [1]. The authors studied ratchet-like structures of the potential, written for $\beta = 0$, for SQUID's with three identical junctions, two in one branch and one in the other. In order to break spatial symmetry in a SQUID, one can also consider a two-junction interferometer circuital model with finite parameter β and with inhomogeneity of the energy coupling of the two junctions [2,3]. In this case, however, it is not generally possible to adopt an effective single-junction description of the system by which a one-dimensional potential can be deduced. Nevertheless, for small finite values of the parameter β and for small structural asymmetries and inhomogeneity in the junctions parameters, a single non-linear first-order ordinary differential equation can still be written to describe the d.c. SQUID behaviour. This result is obtained by a first-order perturbation analysis, taking the parameter β, the structural deformation and the quantities describing inhomogeneity in the junctions as perturbation parameters. It will be shown that this reduced two-junction interferometer model shows additional terms in the effective current phase relation (CPR) of the device, namely, a second-harmonic term due to finiteness of β and a cosine term related to inhomogeneity ε of the energy coupling of the two junctions. It is noted that the critical current of the device, calculated up to first order in the parameters β and ε, however, is not affected by perturbations, except at fields values giving a geometric external flux Φ_{ex} equal to a half integer multiple of the elementary flux quantum Φ_0. In this last case a first order correction in β and ε appears. Finally, for opportune values of the externally applied flux Φ_{ex} and of the two relevant parameters, a ratchet potential with some characteristic features can be displayed.

We start by describing the dynamics of the gauge-invariant superconducting phase differences, φ_1 and φ_2, across the two junctions by means of the Resistively Shunted Junction (RSJ) model [4,5]. Let us assume that the resistive parameters and the maximum Josephson currents of the junctions and the d.c. SQUID branch inductances can be written as follows:

$$R_1 = (1+\delta)\, R, \quad R_2 = (1-\delta)\, R, \tag{1a}$$

$$I_{J1} = (1+\varepsilon)\, I_J, \quad I_{J2} = (1-\varepsilon)\, I_J, \tag{1b}$$

$$L_1 = (1+\lambda)\, L, \quad L_2 = (1-\lambda)\, L, \tag{1c}$$

where δ, ε and λ describe the relative deviations of the model parameters from the corresponding average values R, I_J, and L. In addition, we assume that $\beta = \frac{LI_J}{\Phi_0}$ is itself a perturbation parameter. By writing down the constitutive model equations, we recall that the flux Φ threading the superconducting loop is given by the sum of the external flux Φ_{ex} and the induced flux, so that:

$$\Phi = \Phi_{ex} + L_1 I_1 - L_2 I_2, \tag{2}$$

where I_1 and I_2 are the two branch currents. Moreover, the flux Φ is linked to the gauge-invariant superconducting phase differences through the fluxoid quantization relation

$$\frac{2\pi}{\Phi_0}\Phi + \varphi_1 - \varphi_2 = 2\pi n, \tag{3}$$

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where n is an integer. In this way, the branch currents are related to φ_1 and φ_2 by the following expressions:

$$I_1 = \frac{1}{2}\left[(1-\lambda) I_B - \frac{\Phi_0}{2\pi L}(\varphi_1 - \varphi_2) - \frac{\Phi_{ex}}{L}\right], \quad (4a)$$

$$I_2 = \frac{1}{2}\left[(1+\lambda) I_B + \frac{\Phi_0}{2\pi L}(\varphi_1 - \varphi_2) + \frac{\Phi_{ex}}{L}\right], \quad (4b)$$

where we have taken $n = 0$. By now defining the following normalized quantities

$$i_1 = \frac{I_1}{I_J}, \quad i_2 = \frac{I_2}{I_J}, \quad i_B = \frac{I_B}{I_J},$$
$$\Psi_{ex} = \frac{\Phi_{ex}}{\Phi_0}, \quad \tau = \frac{2\pi R I_J}{\Phi_0} t, \quad (5)$$

and by means of the RSJ model, we can finally write the dynamical equations for the variables φ_1 and φ_2

$$\frac{1}{1+\delta}\frac{d\varphi_1}{d\tau} + (1+\varepsilon)\sin\varphi_1 + \frac{\varphi_1 - \varphi_2}{4\pi\beta} = \frac{1}{2}\left[(1-\lambda) i_B - \frac{\Psi_{ex}}{\beta}\right], \quad (6a)$$

$$\frac{1}{1-\delta}\frac{d\varphi_2}{d\tau} + (1-\varepsilon)\sin\varphi_2 - \frac{\varphi_1 - \varphi_2}{4\pi\beta} = \frac{1}{2}\left[(1+\lambda) i_B + \frac{\Psi_{ex}}{\beta}\right]. \quad (6b)$$

The above coupled non-linear first-order ordinary differential equations represent a rather complete model for describing the SQUID response, recalling, however, that the present analysis does not take into account noise effects, which will be taken into account only at a later stage.

Let us now introduce the following new variables: $\varphi_A = \frac{\varphi_1+\varphi_2}{2}$ and $\Psi = \frac{\varphi_2-\varphi_1}{2\pi}$, which represent the average phase difference and the flux number $\frac{\Phi}{\Phi_0}$, respectively. In terms of these new variables, the dynamical equations are written as follows:

$$\frac{d\varphi_A}{d\tau} + \cos(\pi\Psi)\sin\varphi_A - (\delta+\varepsilon)\sin(\pi\Psi)\cos\varphi_A - \frac{\delta}{2\beta}\Psi = \frac{i_B}{2} - \frac{\delta}{2\beta}\Psi_{ex}, \quad (7a)$$

$$\pi\frac{d\Psi}{d\tau} + \sin(\pi\Psi)\cos\varphi_A - (\delta+\varepsilon)\cos(\pi\Psi)\sin\varphi_A + \frac{\Psi}{2\beta} = (\lambda-\delta)\frac{i_B}{2} + \frac{\Psi_{ex}}{2\beta}. \quad (7b)$$

As said before, for a symmetric and homogeneous SQUID with $\beta = 0$, the above set of equations reduces to the following

$$\frac{d\varphi_A}{d\tau} + \cos(\pi\Psi_{ex})\sin\varphi_A = \frac{i_B}{2}, \quad (8)$$

since, in this case, $\Psi = \Psi_{ex}$ and all other perturbation parameters are null. Therefore, on the basis of this result, we can assume that there exists a perturbed solution of the set of equations (7a-b) of the form

$$\Psi(\tau) = \Psi_{ex} + \beta\Psi_\beta(\tau) + \delta\Psi_\delta(\tau) + \varepsilon\Psi_\varepsilon(\tau) + \lambda\Psi_\lambda(\tau). \quad (9)$$

By substituting equation (9) into equation (7b) and by equating to zero the coefficients of the parameters β, δ, ε, and λ, we get the following expression for $\Psi(\tau)$:

$$\Psi(\tau) = \Psi_{ex} - 2\beta\sin(\pi\Psi_{ex})\cos\varphi_A. \quad (10)$$

Substituting now the above expression in equation (7a) we have:

$$\frac{d\varphi_A}{d\tau} + \cos(\pi\Psi_{ex})\sin\varphi_A + \pi\beta\sin^2(\pi\Psi_{ex})\sin 2\varphi_A - \varepsilon\sin(\pi\Psi_{ex})\cos\varphi_A = \frac{i_B}{2}, \quad (11)$$

which represents a reduced two-junction interferometer model. In equation (11) we notice the appearance of two additional terms in the effective CPR. The first is a second-harmonic sine term, the second a cosine term in φ_A.

It is not difficult to show, by a rather standard analysis, that, for $\Psi_{ex} \neq \frac{2k+1}{2}$, k integer, the normalized critical current of the device is still given by the usual expression $i_c = 2|\cos(\pi\Psi_{ex})|$. However, for $\Psi_{ex} = \frac{2k+1}{2}$, k being an integer, the same analysis carried out for $\varepsilon = 0$, gives $i_c = 2\pi\beta$; for $\beta = 0$, on the other hand, one has $i_c = 2|\varepsilon|$. Therefore, according to the present analysis, the zero-th order model of d.c. SQUID's (Eq. (8)) is able to capture the basic features of the complete model, relying exclusively on the dynamics of the variable φ_A. However, with a first order analysis the dynamical properties of the model arising from the time-variation of the variable Ψcan be approximately taken into account with little effort. Moreover, by the resulting first-order model, one can argue that second order corrections in the perturbation parameters play an important role in d.c. SQUID models, since first-order corrections are present only for $\Psi_{ex} = \frac{2k+1}{2}$, k integer.

We now turn to analyze the ratchet-like potential arising from this model. Setting $x = \varphi_A$, we write equation (11) in the following form:

$$\frac{dx}{d\tau} = -\frac{dU}{dx} + \frac{i_B}{2}, \quad (12)$$

where

$$U(x) = -\cos(\pi\Psi_{ex})\cos x - \frac{\pi\beta}{2}\sin^2(\pi\Psi_{ex})\cos 2x - \varepsilon\sin(\pi\Psi_{ex})\sin x + \text{const.}, \quad (13)$$

is the potential. In Figure 1 we report this potential as a function of the variable x (the average superconducting phase difference) for $\beta = 0.1$, $\varepsilon = 0.25$ and $\Psi_{ex} = 0.35$. We notice a ratchet-like structure of the potential similar

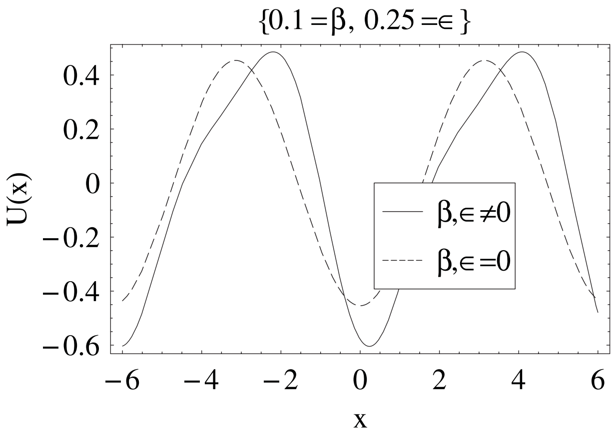

Fig. 1. Ratchet-like potential $U(x)$ (full line) arising from the proposed model and calculated for $\beta = 0.1$, $\varepsilon = 0.25$ and $\Psi_{ex} = 0.35$. Unperturbed potential (dashed line) for a d.c. SQUID ($\beta = 0$, $\varepsilon = 0$, $\Psi_{ex} = 0.35$).

to that reported in reference [1], this time given not by the presence of two junctions in a branch, but rather by a combination of two perturbation effects in the system: a finite β value and a slight inhomogeneity in the junction superconducting coupling energy. Finally notice that the bias current term can be taken as an oscillatory function and that an additive forcing term simulating thermal noise in the system can be added on the right hand side of equation (12), in order to complete the analogy with the model system in reference [1].

In the present work we have proven that a two-junction interferometer model can be reduced to only one non-linear ordinary differential equation even in the presence of structural asymmetry and inhomogeneity in the junctions parameters and for non-null values of β. This reduction comes about from the notion of the unperturbed solution to the problem and from the definition of a solution for the flux number Ψ depending on the perturbation parameters β, δ, ε, and λ. We have seen that a first-order perturbation analysis provides us with a non-linear ordinary differential equation, where only the parameters β and ε appear. In the effective CPR of the device a second-harmonic additional sine term and a cosine term are present. These additional terms do not affect the value of the critical current of the device to first order in the perturbation parameters, except for field values giving a geometric applied flux equal to half-integer multiples of the elementary flux quantum, i.e. for $\Psi_{ex} = \frac{2k+1}{2}$, k integer. For these particular values of Ψ_{ex} a first order correction to the critical current value is found. Finally, by writing the dynamical equation in terms of an effective potential $U(x)$, x being the average superconducting phase differences of the junctions, a ratchet-like structure for this potential can be found.

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